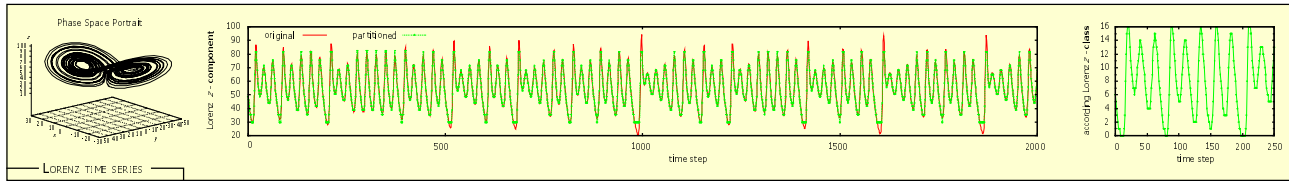


# Modeling Temporal Structure with Neural Methods

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### GRLVQ: generalized relevance learning vector quantization

**CLUSTERING TASK**

**Training Set:**  
Mapping of vectors  $x^i$  to their classes  $y^i$ :  
 $X = \{(x^i, y^i) \in \mathbb{R}^n \times \{1, \dots, C\} \mid i = 1, \dots, m\}$ .

Prototype vectors represent the association of input vectors with its class.  
prototypes:  $W = \{(w^i, y^i) \in \mathbb{R}^n \times \{1, \dots, C\} \mid i = 1, \dots, k \leq m\}$ .

**Prototype Adaptation**

Closest correct prototype  $\tilde{w}^i$ :  
$$\tilde{w}^i := w^i + \epsilon \frac{d^i}{(d_1^i + d_2^i)^2} (x^i - w^i) \quad (1)$$

Closest wrong prototype  $\hat{w}^i$ :  
$$\hat{w}^i := w^i - \epsilon \frac{d_1^i}{(d_1^i + d_2^i)^2} (x^i - w^i) \quad (2)$$

$\epsilon \in (0, 1]$  | learning rate |  
 $f := \text{sigm}(d_1^i - d_2^i)$  | derivative of the sigmoid |  
 $d_1^i = d^i(x^i, \tilde{w}^i)$  | euclidean distance to nearest prototype |  
 $d_2^i = d^i(x^i, \hat{w}^i)$  | euclidean distance to second nearest prototype |  
$$d^i(x^i, w^i) := \sum_{j=1}^n \lambda_j (x_j^i - w_j^i)^2, \quad \lambda_j \geq 0 \quad (3)$$

**Weight Adaptation**

$$\lambda_j := \lambda_j - \epsilon \lambda_j^2 \left( \frac{d_1^i}{(d_1^i + d_2^i)^2} (f - 1) - \frac{d_2^i}{(d_1^i + d_2^i)^2} (f + 1) \right) \quad (4)$$

$\epsilon \in (0, 1]$  | weight adaptation rate |

**Weight normalization:**  $\lambda_j := \lambda_j / |\lambda_j|$ .

**Gradient Descent:**  
Error function:  $A = \sum_{i=1}^m J((x^i - w^i) / (d_1^i + d_2^i))$ .

**Pruning Scheme**  
Set the smallest weight  $\lambda_{j_0} \neq 0$  from a sufficiently trained classification task to constant zero.

**ALGORITHM**

initialize prototype  $(w^i, y^i) \in \mathbb{R}^n \times \{1, \dots, C\}$   
initialize  $\lambda_1 = \dots = \lambda_n = 1/\epsilon$ .

repeat "training":  
choose training example  $(x^i, y^i)$   
compute  $\tilde{w}^i$  and  $\hat{w}^i$  using metric (3),  $d^i(w^i, x^i)$ ,  $y^i$ :  $d^i = d^i$   
adapt both prototypes according to (1) and (2) respectively  
adapt each  $\lambda_j \neq 0$  using (4)  
set  $\lambda_{j_0} := 0$  if  $0 < \lambda_{j_0} < \lambda_{j_0}^2$   
normalize  $\lambda$   
with classification error acceptable.

repeat "pruning":  
apply pruning scheme  
retrain training  
with classification error degrades significantly.

**ILLUSTRATION**

x - class 1 data  
o - class 2 data  
□ - class 1 prototypes  
■ - class 2 prototype(s)

sample:  $(x^i, y^i = 2)$

**DATA PROCESSING**

training sample  $(x^i, y^i)$

time series:  $x_1^i, x_2^i, x_3^i, \dots, x_n^i$

class:  $(y, y+1)$

relevance vector:  $\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_n$

### BASIC IDEA

Taking into consideration the overall *significance* of components for one-step prediction, we formulate an alternative to the delay embedding strategy: input weighting obtained from a recent neural algorithm is used to construct vectors with the time series components of most important relevance.

We apply an adaptation of *Kohonen's* vector quantization scheme (LVQ) with built-in detection of significant input vector components, the **generalized relevance learning vector quantization** (GRLVQ), in order to achieve a relevance based embedding of scalar time series.

Using the neural classification method on time series data we obtain:

(1) *analysis* - temporal properties of a window on the past,  
(2) *modeling* - short-term and long-term prediction.

We use artificial data from a chaotic Lorenz System and real-life runoff data of a long-term time series from a large lysimeter.

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